

Условие

Вычислить интеграл $\int_{(0,0)}^{(1,1)} xy dx + (y-x) dy$ вдоль линии 1) $y = x$, 2) $y = x^2$, 3) $y^2 = x$, 4) $y = x^3$.

Решение

$$1) \int_{(0,0)}^{(1,1)} xy dx + (y-x) dy = \left[\begin{array}{l} y = x; y' = 1; \\ 0 \leq x \leq 1 \end{array} \right] = \int_0^1 x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}.$$

$$2) \int_{(0,0)}^{(1,1)} xy dx + (y-x) dy = \left[\begin{array}{l} y = x^2; y' = 2x dx; \\ 0 \leq x \leq 1 \end{array} \right] = \int_0^1 (x^3 + (x^2 - x) \cdot 2x) dx = \int_0^1 (3x^3 - 2x^2) dx = \left(\frac{3x^4}{4} - \frac{2x^3}{3} \right) \Big|_0^1 = \frac{1}{12}.$$

3)

$$\int_{(0,0)}^{(1,1)} xy dx + (y-x) dy = \left[\begin{array}{l} x = y^2; x' = 2y dy; \\ 0 \leq y \leq 1 \end{array} \right] = \int_0^1 (y^3 \cdot 2y + y - y^2) dy = \int_0^1 (2y^4 - y^2 + y) dy = \left(\frac{2y^5}{5} - \frac{y^3}{3} + \frac{y^2}{2} \right) \Big|_0^1 = \frac{17}{30}.$$

4)

$$\int_{(0,0)}^{(1,1)} xy dx + (y-x) dy = \left[\begin{array}{l} y = x^3; y' = 3x^2 dx; \\ 0 \leq x \leq 1 \end{array} \right] = \int_0^1 (x^4 + (x^3 - x) \cdot 3x^2) dx = \int_0^1 (3x^5 + x^4 - 3x^3) dx =$$

$$= \left(\frac{x^6}{2} + \frac{x^5}{5} - \frac{3x^4}{4} \right) \Big|_0^1 = -\frac{1}{20}.$$

Ответ: 1) $\frac{1}{3}$, 2) $\frac{1}{12}$, 3) $\frac{17}{30}$, 4) $-\frac{1}{20}$.