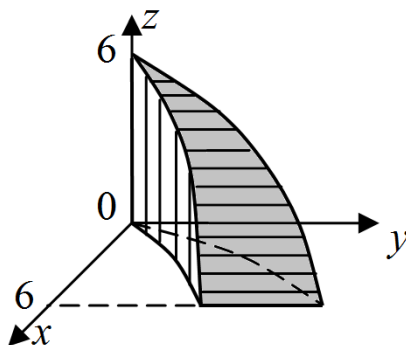


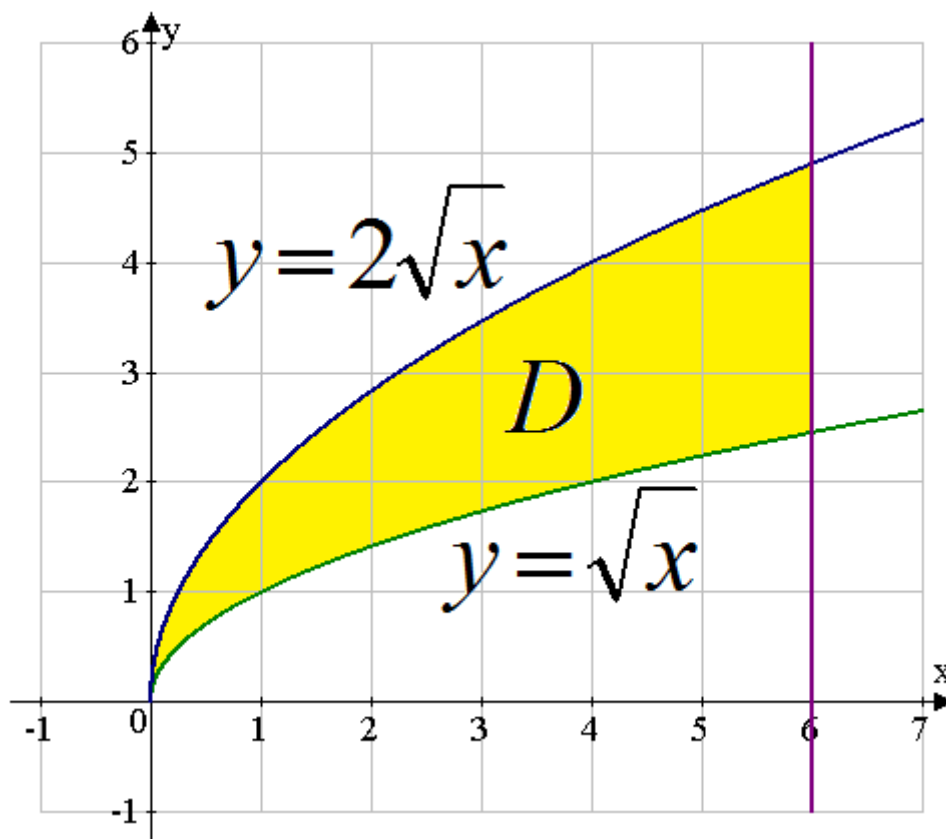
Условие

Найти центр масс однородного тела, ограниченного цилиндрами $y = \sqrt{x}$, $y = 2\sqrt{x}$ и плоскостями $z = 0$, $x + z = 6$.

Решение



Для данного тела имеем $0 \leq z \leq 6 - x$. В проекции на плоскость Oxy получим область D , ограниченную ветвями парабол $y = \sqrt{x}$, $y = 2\sqrt{x}$ и прямой $x = 6$:



Для области D имеем: $0 \leq x \leq 6$, $\sqrt{x} \leq y \leq 2\sqrt{x}$.

$$M = \iiint_G \gamma(x, y, z) dx dy dz = \gamma \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} dy \int_0^{6-x} dz = \gamma \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} z \Big|_0^{6-x} dy = \gamma \cdot \int_0^6 (6-x) dx \int_{\sqrt{x}}^{2\sqrt{x}} dy =$$

$$= \gamma \cdot \int_0^6 (6-x) \cdot y \Big|_{\sqrt{x}}^{2\sqrt{x}} dx = \gamma \cdot \int_0^6 (6-x) \cdot \sqrt{x} dx = \gamma \cdot \int_0^6 \left(6x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx = \gamma \cdot \left(4x^{\frac{3}{2}} - \frac{2x^{\frac{5}{2}}}{5} \right) \Big|_0^6 = \frac{48\gamma\sqrt{6}}{5}$$

$$M_{xy} = \iiint_G z\gamma(x, y, z) dx dy dz = \gamma \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} dy \int_0^{6-x} z dz = \frac{\gamma}{2} \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} z^2 \Big|_0^{6-x} dy = \frac{\gamma}{2} \cdot \int_0^6 (6-x)^2 dx \int_{\sqrt{x}}^{2\sqrt{x}} dy =$$

$$= \frac{\gamma}{2} \cdot \int_0^6 (6-x)^2 \cdot y \Big|_{\sqrt{x}}^{2\sqrt{x}} dx = \frac{\gamma}{2} \cdot \int_0^6 (6-x)^2 \cdot \sqrt{x} dx = \frac{\gamma}{2} \cdot \int_0^6 \left(36x^{\frac{1}{2}} - 12x^{\frac{3}{2}} + x^{\frac{5}{2}} \right) dx = \frac{\gamma}{2} \cdot \left(24x^{\frac{3}{2}} - \frac{24x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{7}{2}}}{7} \right) \Big|_0^6 = \frac{576\gamma\sqrt{6}}{35}.$$

$$M_{xz} = \iiint_G \gamma(x, y, z) y dx dy dz = \gamma \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} y dy \int_0^{6-x} dz = \gamma \cdot \int_0^6 dx \int_{\sqrt{x}}^{2\sqrt{x}} y \cdot z \Big|_0^{6-x} dy = \gamma \cdot \int_0^6 (6-x) dx \int_{\sqrt{x}}^{2\sqrt{x}} y dy =$$

$$= \frac{\gamma}{2} \cdot \int_0^6 (6-x) \cdot y^2 \Big|_{\sqrt{x}}^{2\sqrt{x}} dx = \frac{3\gamma}{2} \cdot \int_0^6 (6-x) \cdot x dx = \frac{3\gamma}{2} \cdot \int_0^6 (6x - x^2) dx = \frac{3\gamma}{2} \cdot \left(3x^2 - \frac{x^3}{3} \right) \Big|_0^6 = 54\gamma.$$

$$M_{yz} = \iiint_G \gamma(x, y, z) x dx dy dz = \gamma \cdot \int_0^6 x dx \int_{\sqrt{x}}^{2\sqrt{x}} dy \int_0^{6-x} dz = \gamma \cdot \int_0^6 x dx \int_{\sqrt{x}}^{2\sqrt{x}} z \Big|_0^{6-x} dy = \gamma \cdot \int_0^6 x(6-x) dx \int_{\sqrt{x}}^{2\sqrt{x}} dy =$$

$$= \gamma \cdot \int_0^6 (6x - x^2) \cdot y \Big|_{\sqrt{x}}^{2\sqrt{x}} dx = \gamma \cdot \int_0^6 (6x - x^2) \cdot \sqrt{x} dx = \gamma \cdot \int_0^6 \left(6x^{\frac{3}{2}} - x^{\frac{5}{2}} \right) dx = \gamma \cdot \left(\frac{12x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{7}{2}}}{7} \right) \Big|_0^6 = \frac{864\gamma\sqrt{6}}{35}.$$

$$x_C = \frac{M_{yz}}{M} = \frac{18}{7}, \quad y_C = \frac{M_{xz}}{M} = \frac{15\sqrt{6}}{16}, \quad z_C = \frac{M_{xy}}{M} = \frac{12}{7}.$$

$$\text{Ответ: } \left(\frac{18}{7}; \frac{15\sqrt{6}}{16}; \frac{12}{7} \right).$$