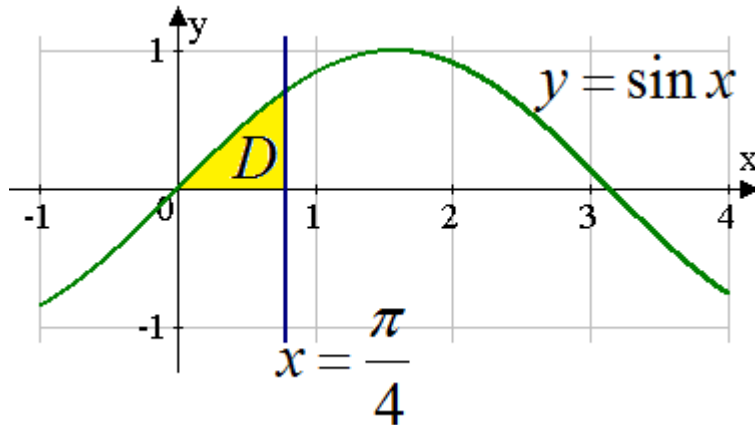


Условие

Двойным интегрированием найти центр масс однородной плоской фигуры, ограниченной синусоидой $y = \sin x$, осью Ox и прямой $x = \frac{\pi}{4}$.

Решение

Для области D имеем: $0 \leq x \leq \frac{\pi}{4}$, $0 \leq y \leq \sin x$. Так как фигура однородна, то плотность $\gamma = \text{const}$.

$$m = \iint_D \gamma(x, y) dx dy = \gamma \cdot \int_0^{\frac{\pi}{4}} dx \int_0^{\sin x} dy = \gamma \cdot \int_0^{\frac{\pi}{4}} y|_0^{\sin x} dx = \gamma \cdot \int_0^{\frac{\pi}{4}} \sin x dx = \gamma \cdot (-\cos x)|_0^{\pi/4} = \gamma \cdot \left(-\frac{\sqrt{2}}{2} + 1 \right).$$

$$M_y = \iint_D x \gamma(x, y) dx dy = \gamma \cdot \int_0^{\frac{\pi}{4}} x dx \int_0^{\sin x} dy = \gamma \cdot \int_0^{\frac{\pi}{4}} x \cdot y|_0^{\sin x} dx = \gamma \cdot \int_0^{\frac{\pi}{4}} x \sin x dx = \left[\begin{array}{l} u = x; \quad du = dx; \\ dv = \sin x dx; \quad v = -\cos x. \end{array} \right] =$$

$$= \gamma \cdot \left(-x \cos x|_0^{\pi/4} + \int_0^{\pi/4} \cos x dx \right) = \gamma \cdot \left(-\frac{\pi\sqrt{2}}{8} + \sin x|_0^{\pi/4} \right) = \gamma \cdot \left(-\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} \right).$$

$$M_x = \iint_D y \gamma(x, y) dx dy = \gamma \cdot \int_0^{\frac{\pi}{4}} dx \int_0^{\sin x} y dy = \frac{\gamma}{2} \cdot \int_0^{\frac{\pi}{4}} y^2|_0^{\sin x} dx = \frac{\gamma}{2} \cdot \int_0^{\frac{\pi}{4}} \sin^2 x dx = \frac{\gamma}{4} \cdot \int_0^{\frac{\pi}{4}} (1 - \cos 2x) dx =$$

$$= \frac{\gamma}{4} \cdot \left(x - \frac{\sin 2x}{2} \right) \Big|_0^{\pi/4} = \frac{\gamma}{4} \cdot \left(\frac{\pi}{4} - \frac{1}{2} \right).$$

$$x_C = \frac{M_y}{m} = \frac{\gamma \cdot \left(-\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} \right)}{\gamma \cdot \left(-\frac{\sqrt{2}}{2} + 1 \right)} = \frac{-\frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2} + 1} = \frac{4 - \pi}{4\sqrt{2} - 4},$$

$$y_C = \frac{M_x}{m} = \frac{\frac{\gamma}{4} \cdot \left(\frac{\pi}{4} - \frac{1}{2} \right)}{\gamma \cdot \left(-\frac{\sqrt{2}}{2} + 1 \right)} = \frac{\pi - 2}{16 - 8\sqrt{2}}.$$

Следовательно, центр масс есть точка $\left(\frac{4-\pi}{4\sqrt{2}-4}; \frac{\pi-2}{16-8\sqrt{2}} \right)$.

Ответ: $\left(\frac{4-\pi}{4\sqrt{2}-4}; \frac{\pi-2}{16-8\sqrt{2}} \right)$.