

Условие

Вычислить интеграл $\iint_D x^2 y \cos(xy^2) dx dy$, $0 \leq x \leq \frac{\pi}{2}$, $0 \leq y \leq 2$.

Решение

$$\begin{aligned} \iint_D x^2 y \cos(xy^2) dx dy &= \int_0^{\frac{\pi}{2}} dx \int_0^2 x^2 y \cos(xy^2) dy = \frac{1}{2} \int_0^{\frac{\pi}{2}} x dx \int_0^2 2xy \cos(xy^2) dy = \frac{1}{2} \int_0^{\frac{\pi}{2}} x dx \int_0^2 \cos(xy^2) d(xy^2) = \\ &= \frac{1}{2} \int_0^{\frac{\pi}{2}} x \cdot \sin(xy^2) \Big|_0^2 dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} x \cdot (\sin(x \cdot 4) - \sin(x \cdot 0)) dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} x \cdot \sin 4x dx = \left[\begin{array}{l} u = x; \quad du = dx; \\ dv = \sin 4x dx; \quad v = -\frac{1}{4} \cos 4x. \end{array} \right] = \\ &= \frac{1}{2} \cdot \left(-\frac{x \cos 4x}{4} \Big|_0^{\pi/2} + \frac{1}{4} \cdot \int_0^{\frac{\pi}{2}} \cos 4x dx \right) = \frac{1}{2} \cdot \left(-\frac{\frac{\pi}{2} \cos(2\pi)}{4} + \frac{1}{16} \cdot \sin 4x \Big|_0^{\pi/2} \right) = \frac{1}{2} \cdot \left(-\frac{\pi}{8} + \frac{1}{16} \cdot 0 \right) = -\frac{\pi}{16}. \end{aligned}$$

Ответ: $-\frac{\pi}{16}$.