

*Условие*Найти интеграл $\int \frac{dx}{\sin 2x + 4 \sin x - 4 \sin^2 x}$.*Решение*

$$\int \frac{dx}{\sin 2x + 4 \sin x - 4 \sin^2 x} = \int \frac{dx}{2 \sin x \cos x + 4 \sin x - 4 \sin^2 x} = \left[\begin{array}{l} t = \operatorname{tg} \frac{x}{2}; \quad dx = \frac{2dt}{1+t^2}. \\ \sin x = \frac{2t}{1+t^2}; \quad \cos x = \frac{1-t^2}{1+t^2}. \end{array} \right] =$$

$$= \int \frac{\frac{2dt}{1+t^2}}{\frac{4t}{1+t^2} \cdot \frac{1-t^2}{1+t^2} + \frac{8t}{1+t^2} - \frac{16t^2}{(1+t^2)^2}} = \int \frac{2(1+t^2)dt}{4t \cdot (1-t^2) + 8t \cdot (1+t^2) - 16t^2} = \int \frac{(1+t^2)dt}{2t^3 - 8t^2 + 6t} = \int \frac{(1+t^2)dt}{2t(t-1)(t-3)}.$$

$$\frac{t^2+1}{t(t-1)(t-3)} = \frac{A}{t} + \frac{B}{t-1} + \frac{C}{t-3} = \frac{A(t-1)(t-3) + Bt(t-3) + Ct(t-1)}{t(t-1)(t-3)}$$

$$t^2+1 = A(t-1)(t-3) + Bt(t-3) + Ct(t-1)$$

$$t=0; \quad 1=3A; \quad A=\frac{1}{3}.$$

$$t=1; \quad 2=-2B; \quad B=-1.$$

$$t=3; \quad 10=6C; \quad C=\frac{5}{3}.$$

$$\frac{t^2+1}{t(t-1)(t-3)} = \frac{1}{3} \cdot \frac{1}{t} - \frac{1}{t-1} + \frac{5}{3} \cdot \frac{1}{t-3}$$

$$\int \frac{(1+t^2)dt}{2t(t-1)(t-3)} = \int \left(\frac{1}{6} \cdot \frac{1}{t} - \frac{1}{2} \cdot \frac{1}{t-1} + \frac{5}{6} \cdot \frac{1}{t-3} \right) dt = \frac{1}{6} \cdot \ln|t| - \frac{1}{2} \cdot \ln|t-1| + \frac{5}{6} \cdot \ln|t-3| + C =$$

$$= \frac{1}{6} \cdot \ln \left| \operatorname{tg} \frac{x}{2} \right| - \frac{1}{2} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{5}{6} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 3 \right| + C$$

$$\text{Ответ: } \int \frac{dx}{\sin 2x + 4 \sin x - 4 \sin^2 x} = \frac{1}{6} \cdot \ln \left| \operatorname{tg} \frac{x}{2} \right| - \frac{1}{2} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 1 \right| + \frac{5}{6} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 3 \right| + C.$$