

*Условие*Найти интеграл $\int \frac{2 \sin x - \cos x}{(3 \sin x + 4 \cos x)^2} dx$.*Решение*

$$\int \frac{2 \sin x - \cos x}{(3 \sin x + 4 \cos x)^2} dx = \left[\begin{array}{l} t = \operatorname{tg} \frac{x}{2}; \quad dx = \frac{2dt}{1+t^2} \\ \sin x = \frac{2t}{1+t^2}; \quad \cos x = \frac{1-t^2}{1+t^2} \end{array} \right] = \int \frac{\left(\frac{4t}{1+t^2} - \frac{1-t^2}{1+t^2} \right) \cdot \frac{2dt}{1+t^2}}{\left(\frac{6t}{1+t^2} + 4 \cdot \frac{1-t^2}{1+t^2} \right)^2} =$$

$$= \int \frac{2 \cdot (4t - 1 + t^2) dt}{(6t + 4 - 4t^2)^2} = \frac{1}{2} \cdot \int \frac{(t^2 + 4t - 1) dt}{(2t^2 - 3t - 2)^2}.$$

$$\frac{t^2 + 4t - 1}{(2t^2 - 3t - 2)^2} = \frac{t^2 + 4t - 1}{(t-2)^2(2t+1)^2} = \frac{A}{t-2} + \frac{B}{(t-2)^2} + \frac{C}{2t+1} + \frac{D}{(2t+1)^2} =$$

$$= \frac{A \cdot (t-2)(2t+1)^2 + B \cdot (2t+1)^2 + C \cdot (t-2)^2(2t+1) + D \cdot (t-2)^2}{(t-2)^2(2t+1)^2}.$$

$$t^2 + 4t - 1 = A \cdot (t-2)(2t+1)^2 + B \cdot (2t+1)^2 + C \cdot (t-2)^2(2t+1) + D \cdot (t-2)^2$$

$$t = 2; \quad 11 = 25B; \quad B = \frac{11}{25}.$$

$$t = -\frac{1}{2}; \quad -\frac{11}{4} = \frac{25D}{4}; \quad D = -\frac{11}{25}.$$

$$t = 0; \quad -1 = -2A + 4C - \frac{33}{25}.$$

$$t = 1; \quad 4 = -9A + 3C + \frac{88}{25}.$$

$$\begin{cases} -A + 2C = \frac{4}{25} \\ -3A + C = \frac{4}{25} \end{cases}$$

$$-A + 2C = -3A + C, \quad 2A = -C, \quad C = -2A.$$

$$-3A - 2A = \frac{4}{25}, \quad A = -\frac{4}{125}, \quad C = \frac{8}{125}.$$

$$\frac{t^2 + 4t - 1}{(2t^2 - 3t - 2)^2} = -\frac{4}{125} \cdot \frac{1}{t-2} + \frac{11}{25} \cdot \frac{1}{(t-2)^2} + \frac{8}{125} \cdot \frac{1}{2t+1} - \frac{11}{25} \cdot \frac{1}{(2t+1)^2}$$

$$\begin{aligned}
& \frac{1}{2} \cdot \int \frac{(t^2 + 4t - 1)dt}{(2t^2 - 3t - 2)^2} = \int \left(-\frac{2}{125} \cdot \frac{1}{t-2} + \frac{11}{50} \cdot \frac{1}{(t-2)^2} + \frac{4}{125} \cdot \frac{1}{2t+1} - \frac{11}{50} \cdot \frac{1}{(2t+1)^2} \right) dt = \\
& = -\frac{2}{125} \cdot \ln|t-2| - \frac{11}{50} \cdot \frac{1}{t-2} + \frac{2}{125} \cdot \ln|2t+1| + \frac{11}{100} \cdot \frac{1}{2t+1} + C = \\
& = -\frac{2}{125} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 2 \right| - \frac{11}{50} \cdot \frac{1}{\operatorname{tg} \frac{x}{2} - 2} + \frac{2}{125} \cdot \ln \left| 2 \operatorname{tg} \frac{x}{2} + 1 \right| + \frac{11}{100} \cdot \frac{1}{2 \operatorname{tg} \frac{x}{2} + 1} + C
\end{aligned}$$

$$\text{Ответ: } \int \frac{2 \sin x - \cos x}{(3 \sin x + 4 \cos x)^2} dx = -\frac{2}{125} \cdot \ln \left| \operatorname{tg} \frac{x}{2} - 2 \right| - \frac{11}{50} \cdot \frac{1}{\operatorname{tg} \frac{x}{2} - 2} + \frac{2}{125} \cdot \ln \left| 2 \operatorname{tg} \frac{x}{2} + 1 \right| + \frac{11}{100} \cdot \frac{1}{2 \operatorname{tg} \frac{x}{2} + 1} + C$$