Основные тригонометрические формулы

Формулы тригонометрических функций суммы и разности двух аргументов

$$\sin(a+b)=\sin a\cos b+\cos a\sin b;\ \sin(a-b)=\sin a\cos b-\cos a\sin b.$$
$$\cos(a+b)=\cos a\cos b-\sin a\sin b;\ \cos(a-b)=\cos a\cos b+\sin a\sin b.$$
$$\operatorname{tg}(a+b)=\frac{\operatorname{tg}a+\operatorname{tg}b}{1-\operatorname{tg}a\operatorname{tg}b};\ \operatorname{tg}(a-b)=\frac{\operatorname{tg}a-\operatorname{tg}b}{1+\operatorname{tg}a\operatorname{tg}b}.$$

Формулы тригонометрических функций двойных, тройных и половинных аргументов

$$\sin 2\alpha=2\sin\alpha\cos\alpha;\ \cos 2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1.$$
$$\operatorname{tg}2\alpha=\frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha};\ \operatorname{ctg}2\alpha=\frac{\operatorname{ctg}^2\alpha-1}{2\operatorname{ctg}\alpha};\ \sin 3\alpha=3\sin\alpha-4\sin^3\alpha;\cos 3\alpha=4\cos^3\alpha-3\cos\alpha.$$
$$\operatorname{tg}3\alpha=\frac{3\operatorname{tg}\alpha-\operatorname{tg}^3\alpha}{1-3\operatorname{tg}^2\alpha};\ \sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2};\ \cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}.$$
$$\operatorname{tg}\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha};\ \operatorname{ctg}\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}.$$

Формулы суммы и разности тригонометрических функций

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2};\ \sin\alpha-\sin\beta=2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$
$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2};\ \cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$
$$\operatorname{tg}\alpha\pm\operatorname{tg}\beta=\frac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta};\ \operatorname{ctg}\alpha\pm\operatorname{ctg}\beta=\frac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}.$$
$$\operatorname{tg}\alpha+\operatorname{ctg}\beta=\frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta};\ \operatorname{tg}\alpha-\operatorname{ctg}\beta=-\frac{\cos(\alpha+\beta)}{\cos\alpha\sin\beta}.$$
$$\cos\alpha+\sin\alpha=\sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right);\ \cos\alpha-\sin\alpha=\sqrt{2}\sin\left(\frac{\pi}{4}-\alpha\right).$$
$$1+\sin\alpha=2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right);\ 1-\sin\alpha=2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right).$$
$$1-\operatorname{tg}^2\alpha=\frac{\cos 2\alpha}{\cos^2\alpha};\ 1-\operatorname{ctg}^2\alpha=-\frac{\cos 2\alpha}{\sin^2\alpha}.$$
$$\operatorname{tg}^2\alpha-\sin^2\alpha=\operatorname{tg}^2\alpha\sin^2\alpha;\ \operatorname{ctg}^2\alpha-\cos^2\alpha=\operatorname{ctg}^2\alpha\cos^2\alpha.$$
$$\operatorname{tg}^2\alpha-\operatorname{tg}^2\beta=\frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cos^2\alpha\cos^2\beta};\ \operatorname{ctg}^2\alpha-\operatorname{ctg}^2\beta=\frac{\sin(\alpha+\beta)\sin(\beta-\alpha)}{\sin^2\alpha\sin^2\beta}.$$

Дополнительные формулы

$$\sin\alpha\sin\beta=\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta));\ \cos\alpha\cos\beta=\frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta));$$
$$\sin\alpha\cos\beta=\frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta)).$$

$$\sin\alpha=\frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}};\ \cos\alpha=\frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}};\ \operatorname{tg}\alpha=\frac{2\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}}.$$

$$\sin^2\alpha+\cos^2\alpha=1;\ \operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1;\ \operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha};\ \operatorname{ctg}\alpha=\frac{1}{\operatorname{tg}\alpha}=\frac{\cos\alpha}{\sin\alpha}.$$

$$\cos^2\alpha=\frac{1}{1+\operatorname{tg}^2\alpha}=\frac{\operatorname{ctg}^2\alpha}{1+\operatorname{ctg}^2\alpha};\ \sin^2\alpha=\frac{1}{1+\operatorname{ctg}^2\alpha}=\frac{\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}.$$

Функции	Углы								
	$-\alpha$	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi k-\alpha$	$2\pi k+\alpha$
sin	$-\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$
cos	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$	$\cos\alpha$
tg	$-\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$	$\operatorname{ctg}\alpha$	$-\operatorname{ctg}\alpha$	$-\operatorname{tg}\alpha$	$\operatorname{tg}\alpha$
ctg	$-\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$	$\operatorname{tg}\alpha$	$-\operatorname{tg}\alpha$	$-\operatorname{ctg}\alpha$	$\operatorname{ctg}\alpha$

α, радианы	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
α, градусы	0	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\operatorname{tg}\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
$\operatorname{ctg}\alpha$	–	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	–

	I четверть	II четверть	III четверть	IV четверть
sin	+	+	–	–
cos	+	–	–	+
tg	+	–	+	–
ctg	+	–	+	–